Math 10B with Professor Stankova
Quiz 10; Tuesday, 4/3/2018
Section #203; Time: 9:30 AM
GSI name: Roy Zhao **Name:** ______________________

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** The partial fraction decomposition of $\frac{1}{(x^2+1)^3}$ is $\frac{A}{x^2+1} + \frac{B}{(x^2+1)^2} + \frac{C}{(x^2+1)^3}$.

   Solution: The numerators should be $Ax + B$, not just constants.

2. **TRUE** False We can use the method of separable equations to solve $r'(s) = e^{r+s}$.

   Solution: We have $e^{r+s} = e^r e^s$ and so we can separate it into a function of r times a function of s so we can use separable equations.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (6 points) Population growth of flowers is given by the differential equation $\frac{dP}{dt} = P(2-P)$. What is the general solution for P? You do not have to explicitly solve for P.

   Solution: This is a separable equation which we solve as

$$\int \frac{dP}{P(2-P)} = \int dt = t + C.$$

   We solve the left side as $\frac{1}{P(2-P)} = \frac{A}{P} + \frac{B}{2-P}$. Multiplying through gives $(2-P)A + BP = 1$ and setting $P = 0, 2$ gives $A = B = \frac{1}{2}$. Therefore

$$\int \frac{dP}{P(2-P)} = \int \frac{1}{2P} + \frac{1}{2(2-P)} dP = \frac{\ln|P|}{2} - \frac{\ln|2-P|}{2} = t + C$$

   is the general solution.

   (b) (2 points) What is the particular solution with the initial condition $P(1) = 1$?

Solution: We just need to plug in the initial condition which gives $(\ln 1)/2 - (\ln 1)/2 = 1 + C = 0$ so $C = -1$ and the solution is $\frac{\ln|P| - \ln|2-P|}{2} = t - 1$.

(c) (2 points) Suppose now that population growth is depends on the season and is given by the differential equation $\frac{dP}{dt} = P(2-P)\cos t$. What is the general solution for P? You do not have to explicitly solve for P. (Hint: try to reuse some of your calculations from part (a))

Solution: This is separable and we separate it as $\frac{dP}{P(2-P)} = \cos t dt$. We integrate the left side and use the calculation from part (a) to get

$$\frac{\ln|P|}{2} - \frac{\ln|2-P|}{2} = \int \cos t dt = \sin t + C.$$